A Theory of Dividend Smoothing[1]

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Abstract

We present a model in which partial smoothing of dividends is an equilibrium outcome. An informed manager who cares about the intrinsic value of the firm but also about the current period's stock price, has to decide how to allocated earnings between investment and dividends. The stock price is determined by uninformed investors. Asymmetric information provides an incentive to inflate dividends and lower investment relative to the first best. We show that a continuum of equilibria exist in which for a given range of earnings the dividend is constant. Hence the variation in dividends does not reflect the full extent of variation in earnings. Compared to the standard separating equilibrium, this partial revelation of earnings induces an investment level that is closer to the first best. Thus, dividend smoothing implies higher firm value, lower average dividends, and lower deviation from first-best investment. We conclude that dividend smoothing provides a partial remedy to underinvestment resulting from information asymmetries. We suggest that last year's dividend can serve as a "focal point," allowing managers and investors to coordinate on just one out of a continuum of equilibria in which dividends are smoothed. Finally, we offer several new testable predictions relating dividend smoothing to investors' mix, managerial incentives, and investment.

1 Introduction

Dividends have long been a puzzle to financial economists (see Allen and Michaely (2002) for an extensive survey). In this paper we focus on one aspect of this puzzle - dividend smoothing. Lintner (1956) interviewed managers from 28 companies and found that they do not set dividends independently in every period. Instead, they first decide whether to change dividends from the existing rate. Managers claimed to reduce dividends only when they had no other choice, and increase dividends only if they were confident that future cash flows can sustain the new dividend level. Two beliefs were expressed strongly: that investors put a premium on companies with stable dividends, and that markets penalize firms that cut dividends. Furthermore, Lintner found that managers were setting the dividend policy first, adjusting other decision cash related decisions to the given dividend level; making dividends one of the main financial decisions of the corporation. Almost fifty years later, in a survey of 384 financial executives, Brav et al. (2005) find that similar considerations still play the dominant role in determining dividends.

While dividend smoothing is an empirical fact, it is quite perplexing theoretically. Why do investors view dividend cuts as bad news in excess of what they have learned from the reported cash flows and earnings? Why don't managers adjust dividends frequently to closer reflect the level of current cash flows? Why are changes in cash flows and earnings reflected in investments and capital structure, but only weakly in dividend changes?

In this paper we attempt to address these questions. We present a theoretical model which shows that dividend smoothing can evolve endogenously as an equilibrium outcome. In our model, the smoothing of dividends forms a partial remedy for the sub-optimal investment problem that arises due to the information asymmetry between managers and investors.

The setup of our model is quite similar to Miller and Rock (1985). An informed manager chooses how to allocate the available cash flows between investments and dividend payout. The total cash flows are private information and cannot be conveyed truthfully to investors, thus the dividend payment serves as a signal. Since shocks to cash flows are correlated across time, today's dividend signals both past and future cash flows. Stock-based compensation makes the manager care about the short term stock price in addition to its long term (intrinsic) value. This provides an incentive for the manager to raise the dividends and underinvest compared to the first best investment policy.

In Miller and Rock (1985) (as well as in many other dividend signaling models) the dividend reveals all the private information of the manager to investors. These perfectly separating (fully revealing) equilibria are very inefficient: the manager overpays and underinvests; and yet he still receives no informational rents. We show that while such a fully revealing equilibrium exists in our setting, it is just one out of a multitude of equilibria. Our focus is on other equilibria which seem to shed light on the economic environment at question.

The equilibria that we consider have full revelation of cash flows for very low and very high outcomes, but for all "intermediate outcomes" the same dividend is announced, i.e., there is a partial pooling of dividends. Thus, for a wide range of cash-flow realizations the manager chooses exactly the same dividend, giving rise to a distribution of dividends that only partially reflects the full variation of cash-flows. In particular, we show the existence of a continuum of equilibria in which the dividend is smoothed with a non-zero probability. These equilibria are somewhat similar to the partially pooling equilibria presented in Harrington (1987) in the context of limit pricing, Bernheim (1994) in the context of conformity, customs and fads, and in particular to Guttman, Kadan and Kandel (2006) in the context of earnings management.

Since both separation and partial smoothing are equilibrium outcomes it is legitimate to ask why smoothing seems to prevail. We show that all of the partially smoothed equilibria Pareto-dominate the fully revealing equilibrium in which no smoothing occurs. Both the manager and the investors benefit from dividend smoothing. First, the manager prefers any of the partially smoothed equilibria over the standard revealing equilibrium, since the investment in the former is closer to the first best. Investors are assumed fully diversified, thus effectively risk neutral. As a result, the stock price reflects, on average, the increased firm value resulting from the higher investment level in the partially smoothed equilibrium.[1] Since all economic agents in our model strictly prefer smoothing over separation it seems likely that smoothing will prevail as an equilibrium.

[1] If investors are risk averse then the Pareto-dominance result becomes more subtle. In that case, partial revelation of information exposes investors to more risk, which has to be weighed against the lower underinvestment.

The multiplicity of equilibria in our model (as in most signaling models) poses a challenge: how can the manager and the investors coordinate on just one out of a continuum of equilibria? While all the partially smoothed equilibria dominate the fully revealing one, we cannot identify the most efficient smoothing equilibrium. We conjecture that coordination on a specific equilibrium is induced exogenously. In particular, it seems that last year's dividend is a natural candidate for a "focal point" that enables investors and managers to coordinate on just one out of a continuum of equilibria. All players coordinate on the smoothed dividend strategy such that the dividend chosen equals the dividend paid in the previous year. Deviations from the last year dividend are observed only if the cash flows are either very low or very high, thus the last year dividend can no longer be supported in a smoothed equilibrium. This is consistent with Lintner's findings and intuition. We illustrate using a detailed example how this coordination argument yields dividend smoothing over time.

To summarize: our argument has three steps. First, we show that in a Miller-Rock-type model there exist a continuum of partially smoothed equilibria. In those equilibria the dividend does not reflect the full variation of changes in cash flows. Second, we show in the world of fully diversified investors *all* the partially smoothed equilibria Pareto-dominate the fully revealing one. Thus, we argue that these are more likely to be observed. Finally, we conjecture that managers and investors choose the last-year' dividend to coordinate on one of these equilibria. This combination predicts that dividends, once announced, persist over time, until the cash flows change to the extent that they no longer support the partially smoothed equilibrium. Then the dividend is cut or increased and the process starts again.

Finally, our model offers several new testable implications. We argue that dividend smoothing will be associated with measures of managerial myopia such as short term stock based compensation. Similarly, the mix of investors' base between short term and long term investors will affect the extent of smoothing. The model also suggests that better investment opportunities will result in more dividend smoothing. Moreover, periods of smoothing will be associated with higher investment and will be followed by periods of high profitability.

The rest of the paper is organized as follows. Section 1.1 discusses related literature. Section 2 develops the basic setup, and presents the separating equilibrium in which no smoothing occurs. In Section 3 we develop and discuss our newly suggested partially smoothed equilibria. Section 4 discusses the efficiency of these equilibria, as well as equilibrium selection and the empirical predictions. Section 5 concludes. Proofs are in the appendices.

1.1 Related Literature

Our model adds to the theoretical literature on dividend signaling.[2] Several papers in this literature relate dividend signaling to the taxation disadvantage that dividends used to have compared to stock repurchases. The first to use this idea is Bhattacharya (1979). In his paper dividends are costly because they are taxed at the ordinary income tax rate, whereas capital gains are taxed at a lower rate. This cost enables managers of good firms to distinguish themselves from bad firms by issuing dividends. John and Williams (1985) also use taxes on dividends as a dissipative cost that facilitate signalling. The idea is that profitable new investments require raising capital which dilutes current shareholders. Managers possess private information about the prospects of the firm. They use dividends to bid up the stock price, and thus decrease the dilution effect of rasing new capital. Signaling equilibria can be sustained since dilution is more damaging to firms with favorable private information. Their model explains why some firms distribute dividends and simultaneously issue new stock, while other firms pay no dividends. Using similar tax-based considerations, Bernheim (1991) presents an equilibrium with pooling at the lower end of the quality spectrum. Thus, firms whose quality falls below a certain threshold may choose not to pay dividends at all. The Jobs and Growth Tax Relief Reconciliation Act of 2003 has diminished the taxation difference between capital gains and dividends, thus may be used to test the tax-based theories (see Poterba, 2004).

Dividend signaling can arise from other sources as well. Miller and Rock (1985) suggest a model in which managers represent both short term and long term stock holders. This partial myopia leads managers to forgo profitable investment and increase dividends trying to boost the short term stock price. Thus, dividend signaling is costly due to the underinvestment problem. Hausch and Seward (1993) study a similar signaling model, and show that the functional form of the firm's production function can impose either deterministic or stochastic dividends. They interpret sto-

[2] The empirical evidence on dividend signaling is mixed. See for instance Penman (1983), Smith and Watts (1992), Bernheim and Wantz (1995), Yoon and Starks (1995), DeAngelo, DeAngelo, and Skinner (1996), Amihud and Murgia (1997), Benartzi, Michaely and Thaler (1997), Nissim and Ziv (2001), Grullon et al. (2005), Johnson et al. (2006), and Chang, Kumar, and Sivaramakrishnan (2006). Allen and Michaely (2002) survey the payout literature.

chastic dividends as stock repurchases. Contrary to Hausch and Seward, dividends in our model are always deterministic, and our results apply for any conventional production function. Ofer and Thakor (1987) develop a model in which the manager chooses between dividend and stock-repurchase signaling. The different cost structure of these two payout methods enables them to explain the stronger price response of stock repurchases. This model, however, does not address the issue of dividend smoothing.

Kumar (1988) presents a signaling model in which managers and investors differ in their level of risk aversion. In his model, dividends serve as a "coordination device" between managers and investors similar to the cheap talk literature originated by Crawford and Sobel (1982). Therefore, in his model, there is no separating equilibrium; the only equilibria are either completely pooling (babbling) or "step-function" equilibria.[3] Our approach is different. We adhere to the standard signaling model of Miller and Rock (1985). We show that previous studies have focused on separating equilibria which form just a subset of the group of equilibria. Moreover, separating equilibria turn out to be highly inefficient compared to equilibria that involve smoothing. This enables us to provide a new rationale for smoothing.

Finally, Allen, Bernardo and Welch (2000) suggest an explanation for dividend smoothing that is different than ours. They argue that differences in taxation between individuals and institutions may explain smoothing. In their model, taxable dividends attract informed institutions whose presence ensures that the firm will remain well run. Dividend reduction will indicate a desire to reduce institutional ownership. So, firms that benefit from institutional ownership will avoid cutting their dividends.

2 Model

2.1 Basic Setup

We develop a two period model in the spirit of Miller and Rock (1985). The cash flows of a firm in period $t = 1, 2$, which are denoted by x_t, are determined by the previous period investment and a random shock:

$$x_t = F(I_{t-1}) + \varepsilon_t,$$

[3] Kumar and Lee (2001) offer a generalization of Kumar's (1988) model to two periods. Here too, the only possible equilibria are step functions and babbling.

where I_{t-1} is the investment in period $t-1$, $F(\cdot)$ is a production function and ε_t is the random shock. The investment in the period prior to the beginning of the game I_0 is given exogenously.

For simplicity, investments are assumed to be non-negative in each period ($I_t \geq 0$).[4] The production function is assumed to be twice continuously differentiable, increasing and concave: $F' > 0$ and $F'' < 0$. We further assume $\lim_{I_t \to 0} F'(I_t) = \infty$ and $\lim_{I_t \to \infty} F'(I_t) = 0$. The random shocks ε_t are drawn from normal distributions with means 0 and variances σ_t^2. Hence x_t is normal with mean $F(I_{t-1})$ and variance σ_t^2 $(t = 1, 2)$. We denote the densities of x_t by g_t and the cumulative distributions of x_t by G_t. The random shocks are assumed uncorrelated: $Cov(\varepsilon_1, \varepsilon_2) = 0$.[5]

At the beginning of Period 1, the manager privately observes the realized cash-flows of the firm, x_1. At times, we shall refer to x_1 as the manager's type. Given his type, the manager decides how to allocate x_1 between dividend payments (D_1) and investment (I_1). Since most of the "action" in the model is in period 1, we will typically suppress the subscripts and use I and D instead of I_1 and D_1. Thus,

$$x_1 = I + D. \tag{1}$$

Investors are risk neutral. They do not observe the random shock ε_1, and hence use the dividend as a signal for the actual cash-flows x_1. All the parameters of the model other then ε_t (and x_t) are common knowledge.

The firm is liquidated in period 2, and shareholders receive a liquidating dividend equal to x_2. Given a Period 1 dividend payment of D and investment of I, the present value of the expected cash flow to the investors (including the dividend) which we refer to as the intrinsic firm value is

$$\begin{aligned} V^I(x_1, D) &= D + \frac{1}{1+i}\left[F(I) + E(\varepsilon_2|\varepsilon_1\right] \\ &= D + \frac{1}{1+i} F(x_1 - D) \end{aligned} \tag{2}$$

where $i \geq 0$ is an appropriate risk-adjusted discount rate. Since investors do not observe ε_1 (and x_1), the manager is the only one who knows the intrinsic value.

The First Best Investment. The first best dividend/investment decision is obtained if the manager seeks to maximize the intrinsic value given by (2). We denote

[4] It is easy to incorporate negative investment (asset sales) into the model.

[5] One can introduce a positive correlation or even a slightly negative one between the random shocks without qualitatively affecting the results in the paper. The previous drat of the paper assumed such correlation.

the first best investment level by I^{FB}: it equates the marginal return on investments with the inter-temporal opportunity cost. That is,

$$F'(I^{FB}) = 1 + i,$$

or

$$I^{FB} = F'^{-1}(1 + i).$$

The Market Value. Since investors observe the dividend, but not the true cash-flows, they use the information contained in the dividend payment to price the stock. Given risk neutrality of the investors, the market value of the firm immediately prior to any known dividend payment at the end of Period 1 is

$$V^M(D) = D + \frac{1}{1+i} E\left(F\left(x_1 - D\right)|D\right). \tag{3}$$

Managerial Remuneration. We assume that the manager is compensated based on both the stock price at period 1 and the liquidating value at period 2. The fact that some of this compensation vests in Period 1, gives rise to some managerial myopia. Thus, instead of maximizing the intrinsic value, the manager chooses dividend/investments to maximize a weighted average of the intrinsic value and short term market value:

$$U(x_1, D) \equiv \alpha V^M(D) + \beta V^I(x_1, D), \tag{4}$$

where $\alpha, \beta > 0$, and V^M and V^I are given by (3) and (2) respectively.

The manager has two conflicting interests: on one hand he would like to boost the stock price by announcing a high dividend resulting in investment that is lower than the first best. On the other hand, he does not want to underinvest too much, because the marginal cost of underinvestment is increasing due to the concavity of $F(\cdot)$. The relative weight that the manager assigns to each of these two objectives is determined by the ratio $\frac{\alpha}{\beta}$. The higher this ratio is, the more inclined is the manager to deviate from the first best investment.

The Basic Trade-offs. To gain better intuition for the trade-offs faced by the manager we will rewrite the manager's utility function in a way that emphasizes the costs and benefits of the dividend/investment decision. First, note that (4) can be written as follows:

$$
\begin{aligned}
U &= \alpha V^M + \beta V^I \\
&= \alpha \left(D + \frac{1}{1+i} E\left(F\left(I\right)|D\right) \right) + \beta \left(D + \frac{F\left(I\right)}{1+i} \right) \\
&= \alpha D + \frac{\alpha}{1+i} E\left(F\left(I|D\right)\right) + \beta \left(D + \frac{F(I)}{1+i} \right).
\end{aligned} \tag{5}
$$

In what follows it is useful to define a separate variable that captures the extent of underinvestment $\Delta \equiv I^{FB} - I$. It is also useful to define a function $h : \mathbb{R} \to \mathbb{R}$ which captures the real cost of underinvestment. That is, $h(\Delta)$ is the difference between the net present value of the first best investment and the NPV of the actual investment, which equals the difference between the intrinsic values under the first best investment and under the actual investment

$$
\begin{aligned}
h\left(\Delta\right) &\equiv \left(\frac{F\left(I^{FB}\right)}{1+i} - I^{FB} \right) - \left(\frac{F\left(I\right)}{1+i} - I \right) \\
&= \frac{1}{1+i} \left(F\left(I^{FB}\right) - F\left(I^{FB} - \Delta\right) \right) - \Delta.
\end{aligned} \tag{6}
$$

Thus, $h(\Delta)$ is a loss function capturing the real loss from underinvestment ignoring any informational effects. It is straightforward to verify that the properties of $F(\cdot)$ imply that $h(\Delta)$ satisfies standard properties of loss functions. In particular: $h(0) = 0$, $h'(0) = 0$; $h'(\Delta) > (<) 0$ iff $\Delta > (<) 0$; $h''(\Delta) > 0$; $\lim_{\Delta \to I^{FB}} h'(\Delta) = \infty$. That is, first-best investment implies no losses, while any deviation from first best investment (positive or negative) implies losses, where the marginal loss is increasing in the deviation from first best.

Using $h(\Delta)$, we can now rewrite the manager's payoff function (5) as follows

$$
U = \alpha D - \beta h(\Delta) + B(D) + C(x_1), \tag{7}
$$

where

$$
\begin{aligned}
B(D) &\equiv \frac{\alpha}{1+i} E\left(F\left(I|D\right)\right) \text{ and} \\
C(x_1) &\equiv \beta \left(x_1 - I^{FB} + \frac{1}{1+i} F\left(I^{FB}\right) \right).
\end{aligned}
$$

The first component, αD, is linear in the paid dividend. It captures the manager's direct benefit from the paid dividend on the stock price, ignoring any informational effects. The second component, $-\beta h(\Delta)$, is the real loss to the manager due to the decrease in intrinsic value resulting from sub-optimal investment. Note that $h(\Delta)$ is independent of investors' beliefs. The third component, $B(D)$, depends on investors' beliefs about the investment given the dividend. Note that $B(D)$ is independent of the actual cash-flows x_1. This component captures the effect of the investors' beliefs on the manager's decision. The last component, $C(x_1)$, depends neither on investors' beliefs nor on the manager's investment decision. It does, however, depend on realized cash-flows x_1.

Thus, the representation in (7) allows us to break down the managers utility into four components: direct price reaction to declared dividend (ignoring informational effects), real cost of suboptimal investment, the effect of investors' beliefs, and a component that depends on the manager's type only.

Equilibrium Definition. A *dividend policy* is a mapping $\Lambda : \mathbb{R} \to \mathbb{R}$ assigning a dividend $D = \Lambda(x_1)$ to any realization of Period 1 cash-flows. Given any dividend D, investors will set beliefs, which are a probability distribution over $\tilde{x}_1$.

A Perfect Bayesian Equilibrium is a combination of a dividend policy and investors' beliefs such that:

1. For all x_1, $\Lambda(x_1) \in \arg\max_D U(x_1, D)$, where the expectations conditional on dividend D are calculated using the investors' beliefs.

2. Investors' beliefs are consistent with $\Lambda(\cdot)$ using Bayes rule, whenever applicable.

2.2 Separating Equilibrium

Observe first that a dividend policy that results in first best investment, i.e., $\forall x_1$ $\Lambda(x_1) = x_1 - I^{FB}$, is not an equilibrium. Indeed, if $\Lambda(x_1) = x_1 - I^{FB}$ for all $x_1 \in \mathbb{R}$, then investors' beliefs must be consistent with this strategy. That is, $B(D)$ in (7) becomes a constant:

$$B(D) \equiv \frac{\alpha}{1+i} F\left(I^{FB}\right).$$

Thus, both $B(D)$ and $C(x_1)$ are independent of the dividend/investment decision. It follows that the manger's choice of dividend is governed by the trade-off between the first two terms in (7): the direct price effect and the real loss due to sub-optimal investment. The marginal benefit from underinvestment (the price effect) is $\alpha > 0$. The marginal cost is $\beta h'(\Delta)$. Since $h'(0) = 0$, the marginal benefit outweighs the marginal cost for a sufficiently small underinvestment level. Thus, given investors' beliefs that the manager invests the first best, all managers' types will find it optimal to deviate and inflate the dividend by lowering the investment below the first best. This implies that the first best cannot be sustained in equilibrium.

In a similar model, Miller and Rock (1985) show the existence of a multitude of separating equilibria. An equilibrium is defined as separating if the manager's type is always revealed given his dividend payment. Our setup differs from Miller and Rock's in that we require a non-bounded support of types. The support in our case is the entire real line. This allows us to concentrate on just one separating equilibrium which induces a linear and therefore tractable dividend policy. This is in contrast to the separating equilibria studied by Miller and Rock which are non-linear and could not be given in a closed form. This tractability is important since it allows us to derive analytically our main results regarding the partially smoothed equilibrium (in the next section). The non-bounded support is crucial because there is no "lowest" type. Such a type does not have an incentive to underinvest, precluding the linearity of equilibrium in setting with a bounded support of types.[6]

To derive the linear separating equilibrium we first conjecture its existence, and then verify that our conjecture is correct. This approach is similar to other papers that derive linear equilibria such as Kyle (1985). Note that first best investment is independent of realized cash-flows. Consequently, we conjecture the existence of a linear equilibrium in which the investment is independent of realized cash flows. Formally, we conjecture the existence of an equilibrium dividend policy of the form

$$\Lambda_s(x_1) = x_1 - I^*, \tag{8}$$

where I^* is a constant level of investment. If the investment is fixed, then investors' beliefs about investment must reflect this and not vary with the dividend. That is, for all D,

$$\begin{aligned} E\left(I|D\right) &= I^* \text{ and} \\ E\left(F\left(I|D\right)\right) &= F(I^*). \end{aligned}$$

This immediately implies that $B\left(D\right)$ in (7) does not vary with D. We denote this constant by

$$B_s \equiv \frac{\alpha}{1+i} F\left(I^*\right). \tag{9}$$

[6] Conceptually, we do not see why our results would not go through also in environments with a bounded support of types. However, we are not able to derive those results analitically due to lack of tractability.

Since $C(x_1)$ also does not depend on D, it follows that the only two components of the manager's utility that play a role in the choice of the dividend are the first two terms in (7).

The actual level of investment given a dividend of D is $I = x_1 - D$, and the level of underinvestment is $\Delta = I^{FB} - (x_1 - D)$. It follows that given the investors' beliefs, the manager's utility (7) can be written as a function of the chosen level of underinvestment as follows:

$$U = \alpha \left(x_1 - (I^{FB} - \Delta)\right) - \beta h(\Delta) + B_s + C(x_1). \tag{10}$$

Note that U is strictly concave in Δ due to the convexity of $h(\cdot)$. Taking the first order condition with respect to Δ we obtain

$$h'(\Delta^*) = \frac{\alpha}{\beta}. \tag{11}$$

Since $h'(0) = 0$ and $\lim_{\Delta \to I^{FB}} h'(\Delta) = \infty$, an interior solution to (11) does exist. The equilibrium underinvestment then satisfies $\Delta^* \in (0, I^{FB})$, implying a strictly positive level of underinvestment.

We thus obtain the following explicit expressions for the equilibrium underinvestment and investment:

$$\begin{aligned} \Delta^* &= h'^{-1}(\alpha/\beta) \text{ and} \\ I^* &= I^{FB} - \Delta^* = I^{FB} - h'^{-1}(\alpha/\beta). \end{aligned} \tag{12}$$

We have completed our argument. We first conjectured the existence of an equilibrium in which dividend policy is linear and investment is constant. We then showed that under investors' beliefs that derive from such a dividend policy it is optimal for the manager to distribute a dividend that keeps investment (and underinvestment) constant.

Given investors' beliefs associated with this separating equilibrium, the market value of the firm following a dividend of D is

$$V^M(D) = D + \frac{1}{1+i} E(F(I)|D) = D + \frac{F(I^*)}{1+i}.$$

The next proposition summarizes these results.

Proposition 1 *The following is a fully separating Perfect Bayesian Equilibrium:*

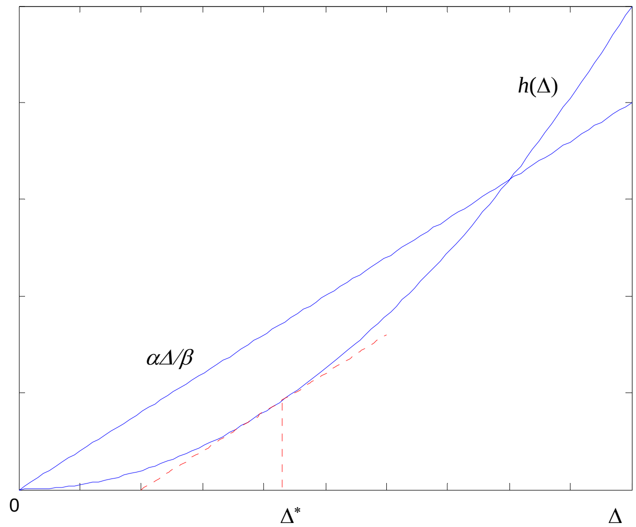

Figure 1: Determinants of Underinvestment in the Separating Equilibrium

1. *Dividend policy is linear and given by* $\Lambda_s(x_1) = x_1 - \left(I^{FB} - \Delta^*\right)$, *where* $\Delta^* = h'^{-1}(\alpha/\beta)$ *is a fixed level of underinvestment.*

2. *Investment is fixed and given by* $I^* = I^{FB} - \Delta^*$.

3. *The dividend reveals the true type of the manager. Given a dividend of* D, *investors assign probability 1 to the event that the manager's type is* $D + I^*$.

4. *The market value of the firm given a dividend of* D *is*

$$V^M(D) = D + \frac{F(I^*)}{1+i}.$$

Figure 1 illustrates this equilibrium. The figure plots the costs and benefits of underinvestment under this equilibrium. The straight line has a slope of α/β and captures the linear benefit of underinvestment due to the inflation in short term price (divided by the constant β). The convex curve is $h(\Delta)$, depicting the real cost of underinvestment. The optimal level of underinvestment Δ^* maximizes the vertical distance between the two curves.

The separating equilibrium possesses some intuitive and simple comparative statics properties that are summarized in the following corollary.

Corollary 1 *In the separating equilibrium:*

1. *Underinvestment increases with α − the extent of stock-based compensation.*
2. *Underinvestment decreases with β - the weight assigned by the manager to the intrinsic value.θ*

The proof is immediate and follows from the fact that $h'(\cdot)$ is an increasing function and hence so is $h'^{-1}(\cdot)$. Intuitively, the manager balances the marginal benefit from a higher short term price with the cost of underinvestment. A higher weight on stock-based compensation (higher $\frac{\alpha}{\beta}$) induces him to manipulate more.

3 Partially Smoothed Equilibrium

The separating equilibrium will serve as a benchmark for our analysis. An important feature of this equilibrium is the fact that different types always pay different dividends. This contradicts the evidence suggesting that managers smooth their dividend payments, and so the variation in dividends does not reflect the full extent of variation in cash-flows or earnings. In the context of this paper we will seek a smoothed-out version of the separating equilibrium: unless cash-flows are extremely small or large - all manger types should pay the same dividend.

We obtained such a partially smoothed dividend policy by constructing an equilibrium in which the dividend varies across types only if the cash-flow realizations are very high or very low. In the proposed equilibrium, the dividend policy is not fully revealing: there is an interval of types $[a, b]$ such that all manager types x_1 in this interval announce the same dividend (the "smoothed dividend"). Outside of this interval, managers follow a separating strategy. As a result, the variation in dividends does not reflect the full extent of variation in cash-flows. Investors' beliefs must be consistent with this smoothed strategy: conditional on observing the smoothed dividend they update their beliefs using Bayes rule given the information that the manager's type falls in the interval $[a, b]$. Thus, while the smoothed dividend provides information to investors by narrowing the set of potential manager's types, this information is not perfect, and the actual type of the manager within the interval is not revealed.

To establish such an equilibrium we start from the linear separating equilibrium derived in the previous section and modify it in a way that yields partial smoothing.

Specifically, let $\delta \geq 0$ be an arbitrary underinvestment level. We construct a continuum of partially smoothed equilibria parametrized by δ. We refer to an equilibrium corresponding to a given δ, as a δ-equilibrium. The dividend policy in each such δ-equilibrium is

$$\Lambda_\delta(x_1) = \begin{cases} b_\delta - \left(I^{FB} - \delta\right) & x_1 \in [a_\delta, b_\delta] \\ \\ \Lambda_s(x_1) & \text{otherwise} \end{cases} \tag{13}$$

That is, in each δ-equilibrium there is a non-empty interval of types $[a_\delta, b_\delta]$ (the "smoothing interval") such that all manager types in this interval announce the same dividend:

$$D_\delta \equiv b_\delta - \left(I^{FB} - \delta\right). \tag{14}$$

The bounds of the interval a_δ and b_δ will be determined endogenously. The smoothed dividend D_δ is the dividend announced by type b_δ (the highest type in the smoothing interval) when this type underinvests by δ. For example, if $\delta = 0$, then all types in the interval $[a_\delta, b_\delta]$ announce a dividend that is the first best dividend of type b_δ.

The investment level made by a type $x_1 \in [a_\delta, b_\delta]$ is then

$$I = x_1 - D_\delta = x_1 - \left(b_\delta - \left(I^{FB} - \delta\right)\right),$$

and the underinvestment made by such a type is $b_\delta - x_1 + \delta$.

All types outside the smoothing interval follow the separating dividend policy: $\Lambda_s(x_1) = x_1 - \left(I^{FB} - \Delta^*\right)$. Figure 2 illustrates the partially smoothed dividend policy for a given δ.

The idea behind this dividend policy is simple. To understand the intuition, consider first the case $\delta = 0$; that is, following the partially smoothed dividend policy, type b_δ invests at the first best level. Managers whose types fall outside of the smoothing interval follow the separating equilibrium. Doing so they reveal their type and underinvest a constant amount of $\Delta^* > 0$. Inside the smoothing interval the story is different. Consider first type $x_1 = b_\delta$: this type must, in equilibrium, be indifferent between the two alternatives - separating and smoothing. If this type chooses the separating dividend he underinvests by Δ^* and the market value of the firm reflects his true type. If, on the other hand, this type chooses the smoothed dividend, he saves the cost of underinvestment since he gets to invest I^{FB}, but then the market price will reflect the fact that investors pool him with lower types in the

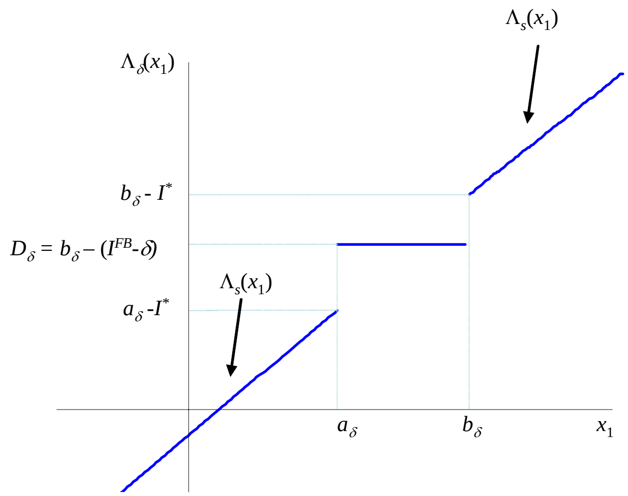

Figure 2: The Partially Smoothed Dividend Policy

interval $[a_\delta, b_\delta]$. The costs and benefits must exactly offset each other for this type, rendering him indifferent.

Consider now type $x_1 = a_\delta$. This type must also be indifferent between the two choices. If he chooses the separating dividend he will underinvest by Δ^* and the market value reflects his true type. If, on the other hand, he chooses the smoothed dividend, the market pools him with higher types in the interval $[a_\delta, b_\delta]$ boosting the stock price compared to the separating alternative. However, under the smoothing strategy he will have to underinvest more.[7] In equilibrium, the two effects must again exactly offset each other rendering type a_δ indifferent.

Similar intuition applies to $\delta > 0$. In a partially smoothed equilibrium type $x_1 = b_\delta$ is pooled with lower types, lowering the market price compared to the separating alternative. To compensate for the decrease in price and render this type indifferent, the smoothed equilibrium's underinvestment must be lower: $\delta < \Delta^*$. Similarly, following a smoothed dividend, type $x_1 = a_\delta$ is pooled with higher types boosting the market price relative to the separating alternative. To render him indifferent, the

[7] Using (14), under the smoothed dividend policy, the underinvestment of type a_δ is

$$I^{FB} - (a_\delta - D_p) = I^{FB} - \left(a_\delta - \left(b_\delta - I^{FB} - \delta\right)\right) = b_\delta - a_\delta + \delta.$$

underinvestment given the smoothed policy must be higher, i.e. $b_\delta - a_\delta + \delta > \Delta^*$.

This shows that a necessary condition for a partially smoothed δ-equilibrium is

$$\Delta^* - (b_\delta - a_\delta) < \delta < \Delta^*. \tag{15}$$

Investors' beliefs must be consistent with the partially smoothed dividend policy (13). Thus, given the normal prior on $\tilde{x}_1$, and observing an announcement of the smoothed dividend $D_\delta = b_\delta - \left(I^{FB} - \delta\right)$, investors will adjust their beliefs using Bayes rule. Hence conditional on a smoothed dividend D_δ, investors' posterior beliefs will be distributed according to a truncated normal distribution over $[a_\delta, b_\delta]$.[8]

Our goal is to show that a continuum of partially smoothed equilibria exist in this model. The first step is to find additional necessary conditions for $\Lambda_\delta(\cdot)$ to be an equilibrium. These necessary conditions stem from the fact mentioned above that being on the border-line, both types $x_1 = a_\delta$ and $x_1 = b_\delta$ must, in equilibrium, be indifferent between following the separating dividend or the smoothed dividend policy.

Using Eq. (10), the utility of type $x_1 = a_\delta$ given a separating dividend is

$$U_s = \alpha\left(a_\delta - I^{FB} + \Delta^*\right) - \beta h\left(\Delta^*\right) + B_s + C(x_1). \tag{16}$$

Since the smoothed dividend is $b_\delta - \left(I^{FB} - \delta\right)$ and the underinvestment given this smoothed dividend for type $x_1 = a_\delta$ is $b_\delta - a_\delta + \delta$, the utility of type $x_1 = a_\delta$ given a smoothed dividend is

$$U_\delta = \alpha\left(b_\delta - I^{FB} + \delta\right) - \beta h\left(b_\delta - a_\delta + \delta\right) + B_\delta + C(x_1),$$

where

$$B_\delta \equiv \frac{\alpha}{1+i} E\left(F\left(I|D_\delta\right)\right) = \frac{\alpha}{1+i}\left(E\left(F\left(x_1 - D_\delta\right)|x_1 \in [a_\delta, b_\delta]\right)\right). \tag{17}$$

By equating U_s and U_δ we obtain the indifference condition for type a_δ:

$$\alpha\left(a_\delta - I^{FB} + \Delta^*\right) - \beta h\left(\Delta^*\right) + B_s = \alpha\left(b_\delta - I^{FB} + \delta\right) - \beta h\left(b_\delta - a_\delta + \delta\right) + B_\delta. \tag{18}$$

Similarly, the indifference condition for type $x_1 = b_\delta$ is

$$\alpha\left(b_\delta - I^{FB} + \Delta^*\right) - \beta h\left(\Delta^*\right) + B_s = \alpha\left(b_\delta - I^{FB} + \delta\right) - \beta h\left(\delta\right) + B_\delta. \tag{19}$$

We established,

[8] This means that for all $q \in [a_\delta, b_\delta]$, the posterior beliefs satisfy $\Pr(\tilde{x}_1 \leq q|\tilde{x}_1 \in [a_\delta, b_\delta]) = \frac{G(q)-G(a_\delta)}{G(b_\delta)-G(a_\delta)}$.

Lemma 1 *A necessary condition for (13) to be a partially smoothed δ-equilibrium dividend policy is that $b_\delta - a_\delta > 0$ and that (18) and (19) are satisfied.*

Subtracting (18) from (19) gives

$$\alpha\left(b_\delta - a_\delta\right) = \beta\left[h(b_\delta - a_\delta + \delta) - h\left(\delta\right)\right]. \tag{20}$$

This simple necessary condition for a δ-equilibrium has an intuitive interpretation: both types a_δ and b_δ must be indifferent between smoothing and separating. In the separating equilibrium there is no difference between the two types in terms of underinvestment - both underinvest by Δ^*. In the partially smoothed δ-equilibrium both types obtain the same price benefit since investors cannot distinguish between them. Since both types are indifferent between smoothing and separating, the difference between the price benefit for the two types in the separating equilibrium which is $\alpha(b_\delta - a_\delta)$, must equal the difference in the cost of underinvestment in the partially smoothed equilibrium which is $\beta\left[h(b_\delta - a_\delta + \delta) - \beta h(\delta)\right]$. We have,

Corollary 2 *A necessary condition for (13) to be a partially smoothed δ-equilibrium dividend policy is that $b_\delta - a_\delta > 0$ and $b_\delta - a_\delta$ satisfies Eq. (20).*

Eq (20) always has one obvious solution: $b_\delta - a_\delta = 0$, which corresponds to the separating equilibrium. The convexity of $h(\cdot)$ implies that there may exist at most one additional solution to this equation, in which $b_\delta - a_\delta$ is strictly positive, and hence smoothing occurs with a positive probability. We claim that there exists a continuum of δ values for which such a nontrivial solution of (20) exists.

For brevity we denote the size of the smoothing interval by $y_\delta \equiv b_\delta - a_\delta$. We claim:

Lemma 2 *There exists a* $0 \leq \underline{\Delta} < \Delta^*$ *such that for all* $\delta \in [\underline{\Delta}, \Delta^*)$ *the equation*

$$\frac{\alpha}{\beta} y_\delta = h(y_\delta + \delta) - h\left(\delta\right). \tag{21}$$

has a unique solution with $y_\delta > 0$. *Moreover, for all* $\delta \in [\underline{\Delta}, \Delta^*)$:

1. $\frac{\partial y_\delta}{\partial \delta} < -1$; *and*

2. *Inequality (15) is satisfied.*

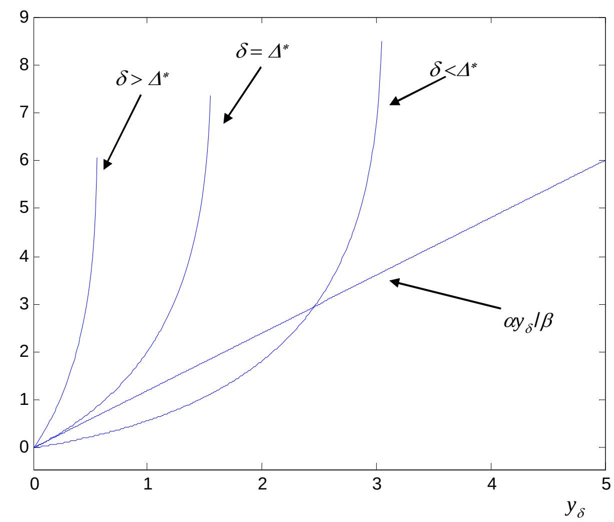

Figure 3: Determinants of the Size of the Smoothing Interval

Figure 3 illustrates the idea behind this lemma and the determinants of the size of the smoothing interval. The figure presents solutions to Eq. (**??**) for three cases: $\delta > \Delta^*$, $\delta = \Delta^*$, and $\delta < \Delta^*$.[9] In the figure we divided both sides by β, so that the straight line is $\alpha y_\delta/\beta$, whereas the three curves depict $h(y_\delta + \delta) - h(\delta)$ for different δ values. When $\delta = \Delta^*$, the slope of the LHS of Eq. (**??**) at $y_\delta = 0$ is exactly equal to the slope of the RHS. Thus, for $\delta = \Delta^*$, the curve $h(y_\delta + \delta) - h(\delta)$ is tangent to the line $\alpha y_\delta/\beta$ at $y_\delta = 0$, and the convexity of the curve implies that $y_\delta = 0$ is the only intersection of the two. When $\delta > \Delta^*$ the slope of the curve at $y_\delta = 0$ is strictly larger than the slope of the line, implying again that $y_\delta = 0$ is the only solution of (**??**). When $\delta < \Delta^*$ but is sufficiently close to Δ^* there is one additional, non-trivial solution to (**??**), represented by the intersection of the two curves. This solution is the size of the smoothing interval. Note that for $\delta < \Delta^*$, as δ becomes smaller the intersection point moves to the right, implying a larger smoothing interval. Moreover, the convexity of the curve implies that the change in the size of the interval is larger than the change in δ : $\frac{\partial y_\delta}{\partial \delta} < -1$.

Given Lemma 2, from now on we restrict attention to $\delta \in (\underline{\Delta}, \Delta^*)$, for which we have shown that a non-trivial smoothing interval with length $y_\delta = b_\delta - a_\delta > 0$ exists.

[9] The figure was plotted using the production function $F(I) = kI^{0.5}$, where k is a scaling factor set to $k = 10$, and a discount factor $i = 0$. Under these assumptions we obtain: $I^{FB} = 4.8$ and $\Delta^* = 3.25$.

For any given such δ The size of the smoothing interval, y_δ, is uniquely determined by (**??**), and does not depend on the location of the interval. Our next task is to make sure that there always exists a location for a smoothing interval $[a_\delta, b_\delta]$ of size y_δ that will render both types a_δ and b_δ indifferent between separating and smoothing. The key here is that investors' beliefs conditional on a smoothing dividend change with the location of the smoothing interval. One has to make sure that there is a location that generates beliefs rendering both types indifferent. The strategy for finding such a location is to search from the left tail to the right tail of the distribution. We show that on the left tail of the distribution the beliefs are such that both types a_δ and b_δ strictly prefer smoothing over separating, while approaching the right tail both strictly prefers separating. Given continuity, there is an intermediate location that makes both indifferent.

Lemma 3 *For each $\delta \in [\underline{\Delta}, \Delta^*)$ there exists an interval $[a_\delta, b_\delta]$ of size y_δ given implicitly by (**??**), such that the necessary conditions (18) and (19) are satisfied. That is, both types a_δ and b_δ are indifferent between the separating and the partially smoothed dividend policies.*

The proof of the lemma is in Appendix B.[10]

So far we focused on necessary conditions only. In particular, the indifference of types a_δ and b_δ between separating and smoothing. In equilibrium much more is needed: all types must (at least weakly) prefer their equilibrium dividend policy over any other dividend. The key to our equilibrium, however, is that the indifference of types a_δ and b_δ together with an appropriate single crossing property and out-of-equilibrium beliefs imply that no other type deviates, establishing sufficiency and existence.

Contrary to the standard signaling literature (e.g. Riley (1979), Miller and Rock (1985)) we cannot use the Spence-Mirrlees single crossing property, which requires calculating the cross derivative of the manager's utility with respect to type and dividend. The problem in using this property is that in the smoothed dividend equi-

[10] This lemma is the only place in the paper where we actually use the properties of the normal distribution. All the other results in the paper only use the non-bounded support of the distribution. We have checked numerically for several examples of production functions and unimodal distributions (from the Elliptic family) and verified that an equilibrium exists. We are not able to show this analytically.

librium we construct, the manager's utility is typically not differentiable in the dividend.[11] Thus, we resort to a more basic single crossing property due to Milgrom and Shannon (1995). This property is satisfied here and does not require differentiability.

Definition 1 *A utility function* $U : \mathbb{R}^2 \rightarrow \mathbb{R}$ *satisfies the Milgrom-Shannon Single Crossing Property (SCP) in* (x, D) *if, for all* $x^H > x^L$ *and* $D^H > D^L$*, if* $U(x^L, D^H) \geq U(x^L, D^L)$ *then* $U(x^H, D^H) > U(x^H, D^L)$.

That is, SCP is satisfied if, whenever a low type manager weakly prefers a high dividend over a low dividend, then a high type manager strictly prefers a high dividend over a low dividend. The following lemma is an immediate consequence of the convexity of $h(\cdot)$.

Lemma 4 *For any given investors' beliefs, the manager's utility* $U(x_1, D)$ *satisfies the Milgrom-Shannon SCP in* (x_1, D).

Note now that given a partially smoothed dividend policy $\Lambda_\delta(\cdot)$ (as defined in (13)) the dividend payment never lies in the range $\left(a_\delta - I^*, b_\delta - I^{FB} + \delta\right) \cup \left(b_\delta - I^{FB} + \delta, b_\delta - I^*\right)$. Thus, to completely specify the equilibrium, we must define the out-of-equilibrium beliefs that are associated with these zero probability dividend payments. For simplicity, we assume first that observing such an out-of-equilibrium dividend, investors believe that the manager is "mistakenly" playing according to the separating dividend policy, and they set the stock price accordingly. In Appendix C we characterize the complete set of out-of equilibrium beliefs, show the existence of monotonic out-of equilibrium beliefs that support the δ-equilibria, and discuss the robustness of these out-of equilibrium beliefs vis-a-vis standard refinement concepts, .

We are now ready to state our main existence result:

Proposition 2 *For any* $\delta \in (\underline{\Delta}, \Delta^*)$ *there exists an interval* $[a_\delta, b_\delta]$ *such that* $\Lambda_\delta(\cdot)$ *as given in (13) constitutes a perfect Bayesian* δ*-equilibrium. This partially smoothed equilibrium satisfies the following properties:*

1. *The size of the smoothing interval is given implicitly by*

$$\alpha\left(b_\delta - a_\delta\right) = \beta\left(h(b_\delta - a_\delta + \delta) - h(\delta)\right).$$

[11] The Spence-Mirrlees SCP requires that for all investors' beleifs, $\frac{\partial^2 U(x_1, D)}{\partial x_1 \partial D} > 0$, that is, the marginal benefit the manager gets from dividend payments increases with type. This formulation, however, clearly requires that the mangers' utility be differentiable in D for all investors' beliefs.

2. *Given a dividend of* $D_\delta = b_\delta - \left(I^{FB} - \delta\right)$ *(the smoothed dividend), investors' beliefs are distributed according to a truncated normal distribution over* $[a_\delta, b_\delta]$. *For any other dividend* D, *investors believe that the manager's type is* $D + I^*$.

3. *The market value of the firm given a dividend of* D *is given by*

$$V^M(D) = \begin{cases} D + \frac{1}{1+i} E\left(F\left(I\right)|D_\delta\right) & D = D_\delta \\ D + \frac{1}{1+i} F\left(I^*\right) & \textit{otherwise} \end{cases} \tag{22}$$

The proof of this proposition consists of a simple application of Lemma 3 and the Milgrom-Shannon SCP. It implies that the indifference conditions for types a_δ and b_δ (Eq. (18) and (19)) applied to the smoothing interval found in Lemma 3 are sufficient to ensure that no other type has an incentive to deviate.

Example. To illustrate the partially smoothed equilibrium assume the production function takes the standard Cobb-Douglass form: $F(I) = kI^t$, where $k > 0$ and $0 < t < 1$. Set parameter values to: $k = 10$, $t = 0.2$, $\alpha = 0.7$, $\beta = 1$, $i = 0$, $\mu_1 = 2$, and $\sigma_1 = 0.2$. Then, it is easy to verify that first best investment is $I^{FB} = 2.38$. The investment in the separating equilibrium is $I^* = 1.23$, and the underinvestment in this case is $\Delta^* = 1.15$. Table 1 presents the results for the partially smoothed equilibrium for several $\delta < \Delta^*$ values. For each δ we first calculated the size of the smoothing interval, y_δ, by numerically solving the implicit equation in (**??**). Then, using a simple grid search we found a_δ and b_δ such that $b_\delta - a_\delta = y_\delta$, and the indifference conditions are satisfied. The smoothed dividend, D_δ, is then given by (14). Using the parameters of the distribution function we also calculate the probability of smoothing: $\Pr\left\{[a_\delta, b_\delta]\right\}$.

The case $\delta = 1.15 = \Delta^*$ corresponds to the separating equilibrium, where the smoothing interval vanishes. Smaller values of δ induce a positively lengthed smoothing interval. For instance, in the case $\delta = 0.8$, the size of the smoothing interval is 0.67, which accounts for more than 3 standard deviations. Any realization of earnings between 1.59 and 2.26 will induce a smoothed dividend of 0.68. The probability that realized cash flows fall in this interval is 0.88. Thus, in this case, the probability of a fully revealing dividend is just 0.12.

In summary, we have completed the first out of the three steps in our argument by showing the existence of a continuum of δ-equilibria. Each one of these equilibria has

	y_δ	a_δ	b_δ	D_δ	$\Pr\{[a_\delta, b_\delta]\}$
$\delta = 1.15$	0	NA	NA	NA	0
$\delta = 1.0$	0.31	1.72	2.03	0.66	0.48
$\delta = 0.9$	0.49	1.67	2.16	0.69	0.74
$\delta = 0.8$	0.67	1.59	2.26	0.68	0.88
$\delta = 0.7$	0.83	1.51	2.34	0.66	0.95
$\delta = 0.6$	0.99	1.42	2.41	0.63	0.98

Table 1: Examples of the Partially Smoothed Equilibrium

dividend smoothing over a positively lengthed interval of size given implicitly by (**??**). Outside of this interval, the dividend policy is fully revealing. We proceed by showing that these equilibria are more "attractive" than the fully revealing equilibrium.

4 Welfare Analysis and Predictions

4.1 Dominance of Smoothing

Having established the partially smoothed equilibria, a legitimate question is why should we consider these equilibria. After all, they are more complicated than the standard separating equilibrium, and they require assumptions on out-of-equilibrium beliefs which are not necessary in the separating equilibrium.

Our answer to this question is two-fold: first, the partially smoothed equilibria offer a chance to explain dividend smoothing, which is a well documented empirical feature of dividend payments, and cannot arise in the standard separating equilibrium. Second, the partially smoothed equilibria provide a simple and tempting explanation for smoothing. We show below that smoothing of dividends is welfare enhancing. It allows managers to underinvest less on average, resulting in higher expected firm value. Both the manager and well diversified (and hence risk neutral) investors, benefit from this increase in value due to the smoothing activity. Thus, dividend smoothing as captured in our new equilibrium is a partial remedy for the underinvestment problem.

Our first result concerns the preferences of the manager between smoothing and separating. It states that the manager strictly prefers any one of the smoothed δ-equilibria over the separating equilibrium. This follows from the fact that no type $x_1 \in (a_\delta, b_\delta)$ has an incentive to deviate from the partially smoothed dividend policy given the out-of-equilibrium beliefs which were set as the separating dividend policy.

Lemma 5 *Let $\delta \in [\underline{\Delta}, \Delta^*)$, and consider a corresponding δ-equilibrium. For every realized cash-flows x_1 the manager weakly prefers to follow the partially smoothed δ-equilibrium over the separating equilibrium. Moreover, for $x_1 \in (a_\delta, b_\delta)$ the manager's preference is strict.*

The next result shows that not only do managers prefer the partially smoothed δ-equilibria, but the value of the firm increases as well. This is so, because the ex-ante firm value is just the average ex-post firm value, conditional on the manager's information. Since the manager is strictly better-off in the partially smoothed equilibrium, it must be that the ex-ante firm value is higher. But, this in turn is possible only if investment moved closer to the first best level. Thus, both the manager and the investors ex-ante prefer any smoothing over perfect separation. Formally,

Proposition 3 *Set $\delta \in [\underline{\Delta}, \Delta^*)$, and consider a corresponding δ-equilibrium. The following holds:*

1. *The expected intrinsic value of the firm is higher under the partially smoothed δ-equilibrium compared to the separating equilibrium.*

2. *The expected underinvestment and expected dividends are lower under the partially smoothed δ-equilibrium when compared to the separating equilibrium.*

3. *The δ-equilibrium generates higher expected investment and higher expected return on investment than the separating equilibrium.*

4. *The δ-equilibrium is ex-ante preferred by the investors over the separating equilibrium.*

5. *The δ-equilibrium Pareto dominates the separating equilibrium.*

Proof: In Appendix A.

Intuitively, smoothing enables the manager to retain some information rents. This is in contrast to the separating equilibrium in which the dividend reveals all of the private information of the manager. The additional uncertainty introduced by the noisy smoothed dividend does not pose a problem to fully diversified (risk neutral) investors, since the firm is still correctly priced on average. The information rents enable the manager to invest more and pay out less, bringing the firm closer to its first best value. Thus, both the manager and the investors benefit from smoothing, hence the Pareto dominance.

This completes the second step of our argument. Any one of the continuum of partially smoothed equilibria Pareto dominates the fully revealing one. This provide some motivation for why a partially smoothed equilibrium will be played by managers and investors (and not the fully revealing equilibrium). Next we discuss how a specific equilibrium may be chosen from the continuum.

4.2 Coordination and Equilibrium Selection

Our theory shows the existence of a continuum of partially smoothed equilibria. How can investors and the manager coordinate on just one out of these equilibria? Our theory is mute regarding this question. In fact, the game theoretic literature in general has very little to say about equilibrium selection. We know that any partially smoothed equilibrium dominates the separating equilibrium. This suggests that managers and investors will try to coordinate on one of the smoothed equilibria. We cannot, however, single out one of the partially smoothed equilibria as the "best" or most efficient one.

Typically, dividend smoothing means that the dividend of one year is likely to be identical to the dividend of the previous year. This suggests that managers and investors use last year's dividend as a "focal point." Thus, they coordinate on playing a partially smoothed $\delta-$equilibrium in which the smoothed dividend D_δ is equal to last year's dividend. In this case, there is an interval of cash-flows such that for all cash-flows in this interval, the manager announce a dividend that equals the last year's dividend. Only if cash-flows are outside this interval, the dividend is different than last year's dividend and then it fully reveals the cash-flows. This seems to fit the intuition in Lintner (1956): managers first decide whether the change in fundamentals is substantial enough to justify a change in dividends, and only if the answer is positive, the dividend is adjusted to reflect actual fundamentals. We note, however, that there is nothing in our theory that necessitates using the last year's dividend as a focal point. The motivation for this argument is, therefore, purely empirical.

We illustrate this argument by an example. In order to introduce dynamics into this example we simply play the game over and over again for several times.[12] Assume

[12] Of course, this exercise is not a real dynamic model. We assume that all of the uncertainty is

that the game is played over 8 years indexed by $n = 1, ..., 8$. The period 1 cash flows in each year are determined by a random walk with a drift:

$$x_{1,n} = 0.1 + x_{1,n-1} + \varepsilon_1,$$

where $\varepsilon_1 \sim N(0, 0.2)$. Thus, in each year n, $x_{1,n}$ is distributed normally, but the mean depends on the realization in the previous year. As a starting point we fix a year 0 earnings as $x_{1,0} = 1.9$ and the year 0 dividend to 0.63. Table 2 presents the results of a path of realizations for the firm earnings. The production function and parameter values are as in Table 1. In particular, $\Delta^* = 1.15$.

Since the realization in year 0 was 1.9, in year 1 earnings are drawn from a normal distribution with mean 2 and s.d. 0.2. We used these parameters to find the δ value such that the smoothed dividend D_δ is equal to the dividend in the previous period - 0.68. The result is $\delta = 0.8$. We assume then, that the manager and the investors choose to play the smoothed equilibrium which corresponds to $\delta = 0.8$. We numerically find the smoothing interval in this case, which turns out to be $[1.59, 2.26]$. Thus, the size of the smoothing interval is $y_\delta = 0.67$ and the probability of smoothing is 0.88. The cash-flow realization in this case is 2.04 which falls inside the smoothing interval. As a result, the dividend announced is $\Lambda_\delta(2.04) = 0.68$, which is equal to the dividend in year 0. Had the separating equilibrium been played, the dividend would have been $\Lambda_s(2.04) = 0.81$. Since the realization in year 1 was 2.04, the year 2 cash-flows are normally distributed with mean 2.14 and s.d. 0.2. Again, we looked for the δ value that will imply $D_\delta = 0.68$ and found it to be $\delta = 1.068$. Again, the assumption is that the manager and investors coordinate on playing this equilibrium. The smoothing interval is now $[1.82, 1.99]$ and the probability of smoothing is 0.18. The cash-flow realization is 1.98 which again falls inside the smoothing interval, implying that the dividend is again $\Lambda_\delta(2.04) = 0.68$. We repeat this process in years 3-8. In year 3 the cash-flow realization falls outside of the smoothing interval for the first time and the dividend is therefore increased to 0.89.

Overall, it is interesting to compare the smoothed dividend column $\Lambda_\delta(x_{1,n})$ with the separating dividend column $\Lambda_s(x_{1,n})$. As opposed to the separating dividend, the variation in the smoothed dividend does not fully reflect the variation in realized cash-flows. Only if realized cash-flows are quite extreme do dividends change.

resolved after each round. It is, however, a useful illustration for how a dividend from a previous period can serve as a focal point for this periods equilibrium selection, and how this implies dividend smoothing.

Year	μ_1	σ_1	δ	y_δ	a_δ	b_δ	$\Pr\{[a_\delta, b_\delta]\}$	D_δ	$x_{1,n}$	$\Lambda_\delta(x_{1,n})$	$\Lambda_s(x_{1,n})$
1	2.00	0.2	0.800	0.67	1.59	2.26	0.88	0.68	2.04	0.68	0.81
2	2.14	0.2	1.068	0.17	1.82	1.99	0.18	0.68	1.98	0.68	0.75
3	2.08	0.2	1.052	0.20	1.81	2.01	0.28	0.68	2.12	0.89	0.89
4	2.22	0.2	0.770	0.72	1.78	2.50	0.91	0.89	2.29	0.89	1.06
5	2.39	0.2	1.081	0.15	2.04	2.19	0.12	0.89	2.35	1.12	1.12
6	2.45	0.2	0.770	0.71	2.02	2.73	0.91	1.12	2.46	1.12	1.23
7	2.56	0.2	1.067	0.17	2.26	2.43	0.20	1.12	2.42	1.12	1.19
8	2.52	0.2	1.041	0.23	2.23	2.46	0.31	1.12	2.52	1.29	1.29

Table 2: An Example of Repeated Play with Coordination on Last Year's Dividend

4.3 Empirical Predictions

Our model has several empirical implications on the relation between dividend smoothing, investments, firm profitability, ownership mix, and managerial incentives.

A first set of empirical implications comes from analyzing the determinants of the size of the smoothing interval given implicitly in (**??**).

Lemma 6 *For any given* $\delta \in [\underline{\Delta}, \Delta^*) : \frac{\partial y_\delta}{\partial \alpha} > 0$, *and* $\frac{\partial y_\delta}{\partial \beta} < 0$.

The proof is immediate since increasing α, and decreasing β increase the slope of the straight line in Figure 3, moving the intersection point further away to the right. Note, however, that an increase in the size of the smoothing interval does not automatically imply a higher probability of smoothing, since the location of the interval changes as well. We have checked numerically (using Cobb-Douglass production functions as in the example above) that the movement of the interval is of second order compared to the increase in size. Thus, the effects of α and β on the size of the interval are reflected in similar effects on the probability of smoothing.

This suggests two empirical predictions. First, managerial myopia is also captured by $\frac{\alpha}{\beta}$. Thus, parameters affecting managerial myopia, such as short term stock based compensation, should be associated with more dividend smoothing. The second prediction stems from a different interpretation of the manager's objective function under which all the results are intact. This interpretation, which is adopted by Miller and Rock (1985), assumes that the manager serves two types of shareholders: short-term and long-term investors. The short term-investors care about the short-term stock price, while the long-term investors care about the long term intrinsic value of the firm. The mix of investors' investment horizon is captured by $\frac{\alpha}{\beta}$. This mix affects the extent of smoothing. Firms with more short-term investors will tend to smooth earnings more.

Next, note that a second way to move the intersection point in Figure 3 to the right and increasing the size of the smoothing interval, is to flatten the convex curve. This curve determines the real cost of underinvestment. A flatter curve means that the marginal cost of underinvestment increases at a slower rate. Our model, thus suggests that better investment opportunities (flatter investment opportunities curve) will be associated with more dividend smoothing.

Finally, Proposition 3 suggests that both investments and return on investment will be higher (on average) when firms smooth dividends. This has both cross-sectional and time-series implications: firms that smooth dividends are expected to invest more, and show higher profitability in following years. Similarly, periods of dividend smoothing should be associated with higher investments, and should be followed by periods of high profitability.

5 Conclusion

In the standard signaling equilibrium (e.g. Miller and Rock (1985), John and Williams (1985)) the dividend reveals all the private information of the manager to the investors. Those equilibria are highly inefficient: the manager overpays and underinvests; and yet he still receives no information rents. We show that while such a fully revealing equilibrium exists in our setting, it is just one out of a multitude of equilibria. The equilibria that we focus on have full revelation of cash-flows for very low and very high outcomes, but for all "intermediate outcomes" the same dividend is announced. Thus, for all cash-flows that fall within a designated interval of cash-flows the manager chooses exactly the same dividend, giving rise to a distribution of dividends that only partially reflects the variation of cash-flows. We show the existence of a continuum of equilibria in which the dividend is smoothed with a non-zero probability.

These partially smoothed equilibria have two appealing features. First, they generate a pattern similar in spirit to the widely documented empirical finding of dividend smoothing. The standard signaling equilibrium does not generate any smoothing, while in cheap talk equilibria the dividends are always smoothed.

Second, our model provides a rationale for dividend smoothing: all equilibria with dividend smoothing allow the manager to underinvest less (on average). This increases firm value and as a result, any equilibrium with smoothed dividends dominates the standard separating equilibrium. The fact that both investors and the manager prefer smoothing over separating suggests that they both may try to coordinate and play this partially smoothed equilibrium. Last year's dividend can then serve as a coordination mechanism allowing them to focus on just one out of a continuum of equilibria.

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Appendix A - Proofs

Proof of Lemma 2: We will show that the straight line $\varphi_1(y) = \frac{\alpha y}{\beta}$ and the curve $\varphi_2(y) = h(y+\delta) - h(\delta)$ must cross at $y > 0$ for some δ values in a left neighborhood of Δ^*.

The slope of $\varphi_2(\cdot)$ is α/β while the slope of $\varphi_2(\cdot)$ is $\varphi_2'(y) = h'(y+\delta)$. Consider first the case $\delta = \Delta^*$. In this case $\varphi_2'(y) = h'(y+\Delta^*)$, and in particular, $\varphi_2'(0) = h'(\Delta^*) = \frac{\alpha}{\beta}$ (by 11). Since $\varphi_1(0) = \varphi_2(0)$ this means that for $\delta = \Delta^*$, $\varphi_2(\cdot)$ is tangential to $\varphi_1(\cdot)$ at 0. Moreover, since $h(\cdot)$ is convex, $y = 0$ is the only intersection point of the two curves. Hence, in this case $\varphi_2(\cdot)$ lies everywhere above $\varphi_1(\cdot)$.

Consider now the case $\delta = \Delta^* - \varepsilon$, where $\varepsilon > 0$ can be arbitrarily small. Then,

$$\varphi_2'(0) = h'(\Delta^* - \varepsilon) < h'(\Delta^*),$$

due to the convexity of $h(\cdot)$. Since $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$ still coincide at $y = 0$, there is a right neighborhood of $y = 0$ for which $\varphi_2(y) < \varphi_1(y)$. Also, because for $\delta = \Delta^*$, $\varphi_2(\cdot)$ lied everywhere above $\varphi_1(\cdot)$ and strictly so for $y > 0$, and since $\varepsilon > 0$ can be arbitrarily small, there must by a $y > 0$ for which $\varphi_2(y) > \varphi_1(y)$. Consequently, by continuity of both $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$, the two curves must meet for some $y > 0$. The crossing point is unique since $h(\cdot)$ is strictly convex.

By the implicit function theorem,

$$\frac{\partial y_\delta}{\partial \delta} = \frac{\beta\left(h'(y_\delta+\delta) - h'(\delta)\right)}{\alpha - \beta h'(y_\delta+\delta)} = -\frac{h'(y_\delta+\delta) - h'(\delta)}{h'(y_\delta+\delta) - h'(\Delta^*)} < -\frac{h'(y_\delta+\delta) - h'(\Delta^*)}{h'(y_\delta+\delta) - h'(\Delta^*)} = -1,$$

where the first equality follows from implicitly differentiating (**??**), the second equality from (11), and the inequality from the convexity of $h(\cdot)$.

Since $y_\delta = b_\delta - a_\delta$, it remains to show that for $\delta < \Delta^*$, $-y_\delta + \Delta^* < \delta$. Define $R(\delta) \equiv \delta + y_\delta - \Delta^*$. Since y_δ vanishes for $\delta = \Delta^*$ we have $R(\Delta^*) = 0$. Moreover,

$$\frac{\partial R(\delta)}{\partial \delta} = 1 + \frac{\partial y_\delta}{\partial \delta} < 0,$$

showing that $R(\delta)$ is strictly positive for $\delta < \Delta^*$ as required. ■

Proof of Lemma 4: Fix investors' beliefs and let $D^H > D^L$. From (7), and recalling that $\Delta = I^{FB} - (x_1 - D)$ we have

$$\begin{aligned} U\left(x_1, D^H\right) - U\left(x_1, D^L\right) &= \alpha\left(D^H - D^L\right) \\ &\quad -\beta\left(h(I^{FB} - x_1 + D^H) - h(I^{FB} - x_1 + D^L)\right) + B(D^H) - B(D^L). \end{aligned}$$

To establish the SCP it is sufficient to show that $\frac{\partial}{\partial x_1}\left(U\left(x_1, D^H\right) - U\left(x_1, D^L\right)\right) > 0$. Differentiating by x_1 gives

$$\frac{\partial}{\partial x_1}\left(U\left(x_1, D^H\right) - U\left(x_1, D^L\right)\right) = \beta\left(h'(I^{FB} - x_1 + D^H) - h'(I^{FB} - x_1 + D^L)\right),$$

which is positive by the convexity of $h(\cdot)$. ■

Proof of Proposition 2: Let $\delta \in (\underline{\Delta}, \Delta^*)$. From Lemma 3 there exists a non empty interval $[a_\delta, b_\delta]$ such that the indifference conditions (18) and (19) are satisfied. We will show that given the indifference and the assumed out-of-equilibrium beliefs, no type has an incentive to deviate.

Given the assumed out-of-equilibrium beliefs, it is straightforward that following any realized cash-flows, announcing a dividend according to the separating strategy of Proposition 1 strictly dominates all other dividends, except the pooling dividend. In particular, following cash-flows of a_δ and of b_δ, the manager who is indifferent between the separating and pooling dividends, does not have an incentive to deviate to any other dividend.

We first show that following any cash-flows $x_1 \in (a_\delta, b_\delta)$ the manager strictly prefers to announce the smoothed dividend, $D_\delta = b_\delta - I^{FB} + \delta$, over any other out–of-equilibrium dividend, $D \in \left(a_\delta - I^{FB} + \Delta^*, b_\delta - I^{FB} + \delta\right) \cup \left(b_\delta - I^{FB} + \delta, b_\delta - I^{FB} + \Delta^*\right)$.

Consider first $D' \in \left(a_\delta - I^{FB} + \Delta^*, b_\delta - I^{FB} + \delta\right)$.

Following cash-flows of a_δ, the manager strictly prefers the smoothed dividend over D'. By the single crossing property, following every cash-flows $x_1' > a_\delta$ the manger strictly prefers the smoothed dividend over D'.

Consider now $D'' \in \left(b - I^{FB} + \delta, b - I^{FB} + \Delta^*\right)$. We assume (by contradiction) that there exists realized cash-flows $x_1'' \in (a_\delta, b_\delta)$ for which the manager strictly prefers the dividend D'' over D_δ. According to the single crossing property, it must be that also following cash-flows b_δ (which are higher than x_1'') the manager strictly prefers the dividend D'' over D_δ. This contradicts the fact that following cash-flows b_δ the manger prefers D_δ over any other dividend.

We now claim that given cash-flows $x_1 \notin (a_\delta, b_\delta)$, type x_1 prefers the separating dividend over the smoothed dividend. This follows from a similar application of the SCP. ■

Proof of Lemma 5: For $x_1 < a_\delta$ and $x_2 > b_\delta$ the smoothed and separating dividend policies are identical implying indifference. For $x_1 = a_\delta$ and $x_2 = b_\delta$ we again have indifference by (18) and (19). Finally, consider $x_1 \in (a_\delta, b_\delta)$. The smoothed dividend policy is to pay a dividend of $D_\delta = \Lambda_\delta(x_1) = b_\delta - \left(I^{FB} - \delta\right)$. The separating dividend policy is to pay a dividend equal to $D_s = \Lambda_s(x_1) = x_1 - \left(I^{FB} - \Delta^*\right)$. Consider the following two cases:

Case 1: $D_\delta \geq D_s$. We know that type a_δ prefers paying D_δ over $\Lambda_s(x_1)$.[13] By the SCP, and since $D_\delta > D_s$ it follows that type x_1 (which is higher than a_δ) strictly prefers the smoothed dividend over the separating dividend policy.

Case 2: $D_\delta < D_s$. Again, we know that type b_δ prefers paying D_δ over $\Lambda_s(x_1)$. By the SCP, and since $D_\delta > D_s$ it follows that type x_1 (which is lower than b_δ) strictly prefers the smoothed dividend over the separating dividend policy. ■

Proof of Proposition 3. Set $\delta \in (\underline{\Delta}, \Delta^*)$.

Claim 1. From Lemma 5 we know that the manager prefers the δ-equilibrium over the separating equilibrium for each and every x_1, and strictly so on a positively lengthed interval. It follows that the manager also strictly prefers the δ-equilibrium over the separating equilibrium in expectation (ex-ante). That is,

$$E_\delta U = E_\delta\left(\alpha V^M\right) + E_\delta\left(\beta V^I\right) > E_s U = E_s\left(\alpha V^M\right) + E_s\left(\beta V^I\right), \tag{23}$$

where E_δ and E_s stand for expectation under the partially smoothed and separating equilibria respectively. On average, the investors price the stock correctly, and hence, in equilibrium the ex-ante expected market value equals the intrinsic value of the firm.[14] That is, $E_\delta\left(V^M\right) = E_\delta\left(V^I\right)$ and $E_s\left(V^M\right) = E_s\left(V^I\right)$. Hence, (23) can be written as

$$(\alpha + \beta) E_\delta(V^I) > (\alpha + \beta)\, E_s(V^I),$$

which implies that $E_\delta(V^I) > E_s(V^I)$. Namely, the expected intrinsic value of the firm is higher given the partially smoothed equilibrium.

Claims 2,3. Rearranging terms in (18) gives

$$\alpha\left(\Delta^* - \delta\right) - \beta\left(h\left(\Delta^*\right) - h\left(\delta\right)\right) = B_\delta - B_s.$$

[13] This is because paying D_δ gives him the same utility as his own separating dividend (by (18)), and his separating dividend is preferred by him to type x_1's separating dividend, since the separating dividend policy is an equilibrium.

[14] Under the separating equilibrium, since there is full revelation of the actual earnings and investment, the stock price always equals the intrinsic value. Under the $\delta-$equilibrium, the market price equals the stock price on average (ex-ante).

By Lemma 10 (in Appendix B), the LHS is strictly positive. It follows that $B_\delta - B_s > 0$.

From the definitions of B_s and B_δ (Eq. (9) and (17)) we have

$$0 < B_\delta - B_s = \frac{\alpha}{1+i} E\left(F\left(I|D_\delta\right)\right) - F\left(I^*\right).$$

This implies that $E\left(F\left(I|D_\delta\right)\right) > F\left(I^*\right)$. Now, the monotonicity and concavity of $F(\cdot)$ imply that also $E\left(I|D_\delta\right) > I^*$.[15] Thus, expected investment and expected return on investment are higher in the δ-equilibrium, and expected underinvestment, and expected dividends are lower.

Claim 4. Since the expected value of the firm is higher under the partially separating equilibrium, and the investors price the stock correctly on average, they ex-ante prefer the partially smoothed δ-equilibrium.

Claim 5. Since both the manager and the investors ex-ante prefer the partially smoothed δ-equilibrium, it Pareto dominates the separating equilibrium. ■

Appendix B - Proof of Lemma 3

In this appendix we provide a proof for Lemma 3. This proof is somewhat lengthy and technical. This is also the only place in the paper where we use the properties of the normal distribution. To streamline the reading we first provide an intuitive sketch of the proof and only then present the formal arguments (which can be skipped in a first reading).

Intuitive Sketch of the Proof of Lemma 3

Given that $\delta \in \left(\underline{\Delta}, \Delta^*\right)$, Eq. (20) uniquely determines the size of the smoothing interval $y_\delta = b_\delta - a_\delta > 0$. The question is whether one can find an interval $[a_\delta, b_\delta]$ of size y_δ such that both types a_δ and b_δ will be indifferent between separating and smoothing. The idea of the proof is to show that if the interval $[a_\delta, b_\delta]$ approaches the left tail of the distribution then both types a_δ and b_δ strictly prefer smoothing over separating, whereas if the interval approaches the right tail of distribution then

[15] To see this assume on the contrary that $E\left(I_1|D_\delta\right) \leq I^*$. Then, by the monotonicity of $F(\cdot)$, we have

$$F\left(E\left(I|D_\delta\right)\right) \leq F\left(I^*\right).$$

And, by Jensen's inequality using the concavity of $F(\cdot)$, $E\left(F\left(I|D_\delta\right)\right) \leq F\left(I^*\right)$ - a contradiction.

both will prefer separating over smoothing. By continuity, there is a location for the interval that makes both a_δ and b_δ exactly indifferent.

To see this, note that under the normal distribution, if an interval of size y_δ lies on the far left tail of the distribution (approaching $-\infty$) then posterior beliefs converge to a distribution that puts a mass of 1 on type b_δ and 0 on all other types in the interval $[a_\delta, b_\delta)$. This is the case because under the normal distribution, relative likelihood ratios become infinitely large when one approaches the left tail. That is, if the interval $[a_\delta, b_\delta]$ is located on the far left, investors, observing the smoothed dividend of D_δ, are arbitrarily close to a perfect information case in which they know that the manager's type is b_δ. Now, think about type b_δ. If he chooses the separating dividend he has to underinvest by Δ^*, and the market price reflects his true type. If he chooses the smoothed dividend, he underinvests by δ, which is strictly smaller than Δ^*, and still the market price reflects his true type (or at least arbitrarily close to this). This implies that type b_δ strictly prefers the smoothed dividend over the separating one. A similar argument applies to type a_δ.

Consider now the other extreme case in which the interval of size y_δ approaches the right tail of the distribution (approaching $+\infty$). The normal distribution implies that investors beliefs following a smoothed dividend of D_δ are approaching perfect knowledge that the manager's type is a_δ. If type b_δ follows the smoothed dividend then the stock price reflects beliefs that his type is a_δ (or arbitrarily close to this) and he underinvests by δ. If type b_δ follows the separating dividend then investors price him as type b_δ, but he has to underinvest by Δ^*. The loss in price from smoothing is then $\alpha(b_\delta - a_\delta)$ while the loss in underinvestment from separating is $\beta\left(h(\Delta^*) - h\left(\delta\right)\right)$. But since $\alpha(b_\delta - a_\delta) > \beta\left(h\left(\Delta^*\right) - h\left(\delta\right)\right)$ (see Figure 3), the type b_δ manager strictly prefers separating. A similar argument applies to type a_δ.

From the continuity of the manager's utility in investors' beliefs, it must be that there is an intermediate location in which both types a_δ and b_δ are exactly indifferent between the two alternatives.

Formal Proof of Lemma 3

We first prove some preliminary results which are given in Lemmas 7, 8, 9, and 10 below.

Lemma 7 *For all $y > 0$ the following hold:*

$$\begin{aligned}\lim_{a\to\infty} E\left(x_1 - a | x_1 \in [a, a+y]\right) &= 0\\ \lim_{a\to-\infty} E\left(x_1 - (a+y)\, | x_1 \in [a, a+y]\right) &= 0.\end{aligned}$$

Proof of Lemma 7: Let $y > 0$. Recall that x_1 is normally distributed. Without loss of generality we can assume that this distribution is standard normal (otherwise apply the proof below to the standardized version of x_1). This means that for any x_1,

$$g'(x_1) = -x_1 g(x_1).$$

Hence,

$$\begin{aligned}E\left(x_1 | x_1 \in [a, a+y]\right) &= \frac{\int_a^{a+y} x_1 g(x_1) dx_1}{G(a+y) - G(a)} = \frac{-\int_a^{a+y} g'(x_1) dx_1}{G(a+y) - G(a)}\\ &= -\frac{g\left(a+y\right) - g\left(a\right)}{G(a+y) - G(a)}.\end{aligned}$$

It follows that,

$$\begin{aligned}E\left(x_1 - a | x_1 \in [a, a+y]\right) &= -a - \frac{g(a+y) - g(a)}{G(a+y) - G(a)}\\ &= \frac{-a\left(G(a+y) - G(a)\right) - \left(g(a+y) - g(a)\right)}{G(a+y) - G(a)}.\end{aligned}$$

When a tends to ∞ both the numerator and the denominator tend to 0. Applying L'Hopital's law gives

$$\begin{aligned}\lim_{a\to\infty} E\left(x_1 - a | x_1 \in [a, a+y]\right) &= \frac{-\left(G(a+y) - G(a)\right) + y g(a+y)}{g(a+y) - g(a)}\\ &= \lim_{a\to\infty} \frac{1}{E\left(x_1 | x_1 \in [a, a+y]\right)} + y \lim_{a\to\infty} \frac{g(a+y)}{g(a)} = 0,\end{aligned}$$

since $\lim_{a\to\infty} E\left(x_1 | x_1 \in [a, a+y]\right) = \infty$ and because

$$\lim_{a\to\infty} \frac{g(a+y)}{g(a)} = \lim_{a\to\infty} \frac{\exp\left(-\frac{(a+y)^2}{2}\right)}{\exp\left(-\frac{a^2}{2}\right)} = 0.$$

The second limit is obtained in a parallel manner. ■

Lemma 8 *Let $\delta \in \left(\underline{\Delta}, \Delta^*\right)$, and let $y_\delta > 0$ be the uniquely determined size of the smoothing interval. Also, let $D_\delta = a + y_\delta - \left(I^{FB} - \delta\right)$ be the smoothed dividend corresponding to a and δ. Define*

$$\begin{aligned}L_1\left(a\right) &\equiv E\left(F(x_1 - D_\delta) - F(a - D_\delta) | x_1 \in [a, a+y_\delta]\right)\\ L_2\left(a\right) &\equiv E\left(F(x_1 - D_\delta) - F(a + y_\delta - D_a) | x_1 \in [a, a+y_\delta]\right).\end{aligned}$$

Then, $\lim_{a\to\infty} L_1(a) = 0$ *and* $\lim_{a\to-\infty} L_2(a) = 0$.

Proof of Lemma 8: Using the mean value theorem, we can write

$$L_1(a) = E\left(F'(\xi_a)(x_1 - a)\,|x_1 \in [a, a + y_\delta]\right),$$

where

$$I^{FB} - \delta - y_\delta < \xi_a < x_1 - a - y_\delta + I^{FB} - \delta.$$

Since $x_1 - a < y_\delta$ we have

$$I^{FB} - y_\delta < \xi_a < I^{FB}.$$

That is, regardless of a, ξ_a is bounded in the interval $[I^{FB} - y_\delta, I^{FB}]$. Since $F'(\cdot)$ is continuos it follows that $F'(\xi_a)$ is uniformly bounded. So, there are $\overline{m}, \underline{m} > 0$ such that $\underline{m} \le F'(\xi_a) \le \overline{m}$ for all $a \in \mathbb{R}$. It follows that

$$\underline{m}E(x_1-a)|x_1 \in [a, a+y_\delta] \le E\left(F'(\xi_a)(x_1 - a)|x_1 \in [a, a + y_\delta]\right) \le \overline{m}E(x_1-a)|x_1 \in [a, a+y_\delta].$$

But, from Lemma 7 both the LHS and the RHS of this inequality tend to 0 as $a \to \infty$. This shows that $\lim_{a\to\infty} L_1(a) = 0$. The second limit is proved in a parallel manner.
■

Lemma 9 *Set* $\delta \in [\underline{\Delta}, \Delta^*)$, *and let* $y_\delta > 0$ *be the uniquely determined size of the smoothing interval. Also, let* $D_\delta = a + y_\delta - \left(I^{FB} - \delta\right)$ *be the smoothed dividend. For any* $a \in \mathbb{R}$ *define*

$$J(a) \equiv E\left(F(x_1 - D_\delta) - F(I^*)|x_1 \in [a, a + y_\delta]\right)$$

Then

$$\begin{aligned}
\lim_{a\to\infty} J(a) &= F\left(I^{FB} - \delta - y_\delta\right) - F(I^{FB} - \Delta^*) < 0 \\
\lim_{a\to-\infty} J(a) &= F\left(I^{FB} - \delta\right) - F(I^{FB} - \Delta^*) > 0.
\end{aligned}$$

Proof of Lemma 9:

$$\begin{aligned}
\lim_{a\to\infty} J(a) &= \lim_{a\to\infty} E\left(F(x_1 - D_\delta) - F(I^*)|x_1 \in [a, a + y_\delta]\right) \\
&= \lim_{a\to\infty} E\left(F(x_1 - D_\delta) - F(a - D_\delta)\right)|x_1 \in [a, a + y_\delta] \\
&\quad + \lim_{a\to\infty} E\left(F(a - D_\delta) - F(I^*)|x_1 \in [a, a + y_\delta]\right) \\
&= \lim_{a\to\infty} E\left(F(a - D_\delta) - F(I^*)|x_1 \in [a, a + q]\right) \quad \text{(by Lemma 8)} \\
&= F\left(I^{FB} - \delta - y_\delta\right) - F(I^{FB} - \Delta^*) \quad \text{(using } D_\delta = a + y_\delta - \left(I^{FB} - \delta\right)\text{)}.
\end{aligned}$$

From Lemma 2, $\Delta^* < \delta + y_\delta$. This implies that $F\left(I^{FB} - \delta - y_\delta\right) - F(I^{FB} - \Delta^*) < 0$, as required.

Similarly,

$$\begin{aligned}
\lim_{a \to -\infty} J(a) &= \lim_{a \to -\infty} E\left(F\left(x_1 - D_\delta\right) - F(I^*) | x_1 \in [a, a + y_\delta]\right) \\
&= \lim_{a \to -\infty} E\left(F\left(x_1 - D_\delta\right) - F(a + y_\delta - D_\delta)\right) | x_1 \in [a, a + y_\delta] \\
&\quad + \lim_{a \to -\infty} E\left(F\left(a + y_\delta - D_\delta\right) - F(I^*) | x_1 \in [a, a + y_\delta]\right) \\
&= \lim_{a \to -\infty} E\left(F\left(a + y_\delta - D_\delta\right) - F(I^*) | x_1 \in [a, a + y_\delta]\right) \quad \text{(by Lemma 8)} \\
&= F\left(I^{FB} - \delta\right) - F(I^{FB} - \Delta^*) > 0, \ \text{(using } D_\delta = a + y_\delta - \left(I^{FB} - \delta\right))
\end{aligned}$$

where the inequality follows since $\delta < \Delta^*$. ■

Lemma 10 *For all* $\delta < \Delta^*$, $\alpha\left(\Delta^* - \delta\right) - \beta\left(h\left(\Delta^*\right) - h\left(\delta\right)\right) > 0$.

Proof of Lemma 10: Let $\delta < \Delta^*$. We have

$$\alpha\left(\Delta^* - \delta\right) - \beta\left(h\left(\Delta^*\right) - h\left(\delta\right)\right) = \alpha\Delta^* - \beta h\left(\Delta^*\right) - \left(\alpha\delta - \beta h\left(\delta\right)\right).$$

Recall that Δ^* is the underinvestment level in the separating equilibrium. That is, $\Delta^* = \arg\max_\Delta \left(\alpha\Delta - \beta h\left(\Delta\right)\right)$. This implies that $\alpha\Delta^* - \beta h\left(\Delta^*\right) > \alpha\delta - \beta h\left(\delta\right)$, as required. ■

We turn now to the proof of Lemma 3 itself.

Set $\delta \in (\underline{\Delta}, \Delta^*)$ and let $y_\delta > 0$ be the unique positive solution to Eq. (**??**), which exists by Lemma 2. We will show that there exists an interval $[a_\delta, b_\delta]$ such that $b_\delta = a_\delta + y_\delta$ and the indifference conditions (18) and (19) are satisfied.

Consider first the indifference condition (19). Let $H(a)$ be the difference between the RHS and the LHS of (19):

$$H(a) = B_\delta - B_s - \alpha\left(\Delta^* - \delta\right) + \beta\left(h\left(\Delta^*\right) - h(\delta)\right).$$

Showing that (19) is satisfied is equivalent to showing that there is an a such that $H(a) = 0$.

Using the definitions of B_δ and B_s and Lemmas 9 and 10 we have,

$$\begin{aligned}
\lim_{a \to \infty} H(a) &= -\alpha\left(\Delta^* - \delta\right) + \beta\left(h\left(\Delta^*\right) - h(\delta)\right) \\
&\quad + \frac{\alpha}{1+i} \lim_{a \to \infty} E\left(F\left(x_1 - D_\delta\right) - F(I^*) | x_1 \in [a, a + y_\delta]\right)\right) \\
&= -\alpha\left(\Delta^* - \delta\right) + \beta\left(h\left(\Delta^*\right) - h\left(\delta\right)\right) + \frac{\alpha}{1+i} \lim_{a \to \infty} J(a) < 0.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\lim_{a\to-\infty} H(a) &= -\alpha\left(\Delta^*-\delta\right)+\beta\left(h\left(\Delta^*\right)-h(\delta)\right)+\frac{\alpha}{1+i}\lim_{a\to-\infty} J(a) \\
&= -\alpha\left(\Delta^*-\delta\right)+\beta\left(h\left(\Delta^*\right)-h(\delta)\right) \\
&\quad +\frac{\alpha}{1+i}\left(F\left(I^{FB}-\delta\right)-F(I^{FB}-\Delta^*)\right) \\
&= \beta h\left(\Delta^*\right)+\alpha\underbrace{\left(\frac{1}{1+i}\left(F\left(I^{FB}\right)-F\left(I^{FB}-\Delta^*\right)\right)-\Delta^*\right)}_{Term\ 1} \\
&\quad -\beta h\left(\delta\right)-\alpha\underbrace{\left(\frac{1}{1+i}\left(F\left(I^{FB}\right)-F\left(I^{FB}-\delta\right)\right)-\delta\right)}_{Term\ 2}.
\end{aligned}$$

From the definition of $h(\cdot)$ (Eq. (6)) it follows that Term 1 is equal to $h(\Delta^*)$, and Term 2 is equal to $h(\delta)$. Hence,

$$\lim_{a\to-\infty} H(a)=(\alpha+\beta)\left(h\left(\Delta^*\right)-h\left(\delta\right)\right)>0,$$

where the inequality follows since $0\leq\delta<\Delta^*$ and $h(\cdot)$ is strictly increasing for positive underinvestment levels.

We have shown that $H(a)$ changes signs when we move from a in the left tail to the right tail of the distribution. From continuity, there exists an a such that $H(a)=0$, implying that the indifference condition (19) is satisfied. Choose one such a and denote it by a_δ. Set $b_\delta=a_\delta+y_\delta$. Since both (19) and (20) are satisfied, and since (18) is the difference of (19) and (20) it follows that (18) is satisfied as well. ■

Appendix C - Out-of-Equilibrium Beliefs

The game theoretic literature on equilibrium refinements offers a multitude of concepts to limit the freedom of the modeler in choosing "reasonable" out-of-equilibrium beliefs in signaling games. One prevalent criterion for refinement is the "Intuitive Criterion" of Cho and Kreps (1987). It is straightforward to verify that all the partially smoothed equilibria survive this criterion. In fact, the Intuitive Criterion works best in models with just two types of informed parties, therefore it doesn't impose much restriction on out of equilibrium beliefs in our continuous type framework. Other, stronger criteria used to refine equilibrium with a continuum of types are D1 (Cho and Kreps, 1987), Universal Divinity (Banks and Sobel, 1987), and Never-a-Weak-Best-Response (Kohlberg and Mertens, 1986). Cho and Sobel (1990) show that all of these three are equivalent in the presence of single crossing and other standard properties (satisfied here). Furthermore, they show that these three rule out any kind of pooling in equilibrium. Hence, these criteria cannot be used to refine our partially smoothed equilibria (unless one would like to rule them out up-front).

Our approach in checking the robustness of the out-of-equilibrium beliefs is similar to Harrington (1987). His idea is that out-of-equilibrium beliefs are "reasonable" if the expected type given the signal is increasing in the signal. In our setting, this means that higher dividends should be associated with higher types (on average) both on and off the equilibrium path. Formally,

Definition 2 *We say that investors beliefs are monotone if for all D (both on and off the equilibrium path), $E(x_1|D)$ is non-decreasing in D.*

While deriving the partially smoothed δ-equilibria we assumed for simplicity that if investors observe an out-of-equilibrium dividend then they believe that the manager is "mistakenly" playing the benchmark linear equilibrium. This set of out-of-equilibrium beliefs, while simple and sufficient to support the δ-equilibrium, is not monotone, since out-of-equilibrium dividends that just slightly exceed D_δ entail an expected type lower that the conditional type given D_δ. In the rest of this appendix we show that each δ-equilibrium can be supported by monotone out-of-equilibrium beliefs, maintaining all of the results in the paper (such as the welfare results and the empirical predictions).

Definition 3 *Let $\delta \in [\underline{\Delta}, \Delta^*)$. Consider an out-of-equilibrium dividend $\hat{D} \in \left(a_\delta - I^*, b_\delta - I^{FB} + \delta\right) \cup \left(b_\delta - I^{FB} + \delta, b_\delta - I^*\right)$. We say that beliefs given $\hat{D}$ are concentrated, if there is a type $\hat{x}$ such that the beliefs given $\hat{D}$ assign probability 1 to $\hat{x}$. We say that the out-of-equilibrium beliefs are concentrated if they are concentrated given all $\hat{D} \in \left(a_\delta - I^*, b_\delta - I^{FB} + \delta\right) \cup \left(b_\delta - I^{FB} + \delta, b_\delta - I^*\right)$.*

We will show that there is a concentrated set of out-of-equilibrium beliefs that is also monotone.

The single crossing property suggests that if a_δ is indifferent between its smoothed dividend $D_\delta = b_\delta - I^{FB} + \delta$ and an out-of-equilibrium dividend $\hat{D} \in \left(a_\delta - I^*, b_\delta - I^{FB} + \delta\right)$, then all types in the smoothing interval (a_δ, b_δ) will strictly prefer the smoothed dividend. Similarly, if b_δ is indifferent between the smoothed dividend and and out-of-equilibrium dividend $\hat{D} \in \left(b_\delta - I^{FB} + \delta, b_\delta - I^*\right)$ then all types in the smoothing interval strictly prefer the smoothing dividend. Thus, a natural candidate for a monotone and concentrated set of out-of-equilibrium beliefs is those beliefs that render types a_δ and b_δ indifferent between the smoothed divided and the appropriate out-of-equilibrium dividend.

Let c_δ be implicitly defined by

$$F\left(c_\delta - D_\delta\right) = E\left(F\left(x - D_\delta\right)|D_\delta\right). \tag{24}$$

That is, if investors assign a probability 1 to the event that the manager's type is c_δ, then given a dividend of D_δ the market price would be the same as the market price in the smoothed dividend case. Clearly, $c_\delta \in (a_\delta, b_\delta)$. The uniqueness of c_δ is a simple application of the implicit function theorem.

Consider now an out-of-equilibrium dividend $\hat{D}$ and a concentrated belief $\hat{x} \in (a_\delta, b_\delta)$. The market price given these two is given by

$$V^M\left(\hat{x}, \hat{D}\right) = \hat{D} + \frac{1}{1+i}F\left(\hat{x} - \hat{D}\right).$$

Consider first an out-of-equilibrium dividend $\hat{D} \in \left(a_\delta - I^*, b_\delta - I^{FB} + \delta\right)$. Compare the utility of a type a_δ manager who issues a dividend $\hat{D}$ to the utility of such a manager if he issues the separating dividend. The former naturally depends on the out-of-equilibrium beliefs. If beliefs are such that the market price is identical to the separating price, then the manager obviously prefers the separating dividend over $\hat{D}$ because the market price is identical, while the intrinsic value is lower given $\hat{D}$. If on the other hand, beliefs are such that the market price is equal to $V^M(D_\delta)$ (Eq. 22), then the type a_δ manager strictly prefers $\hat{D}$ over the separating dividend. This is so because the market price is identical now to the smoothed dividend price, while the intrinsic value is higher (lower underinvestment), and we know that a type a_δ is indifferent between separation and smoothing. By continuity, there is a market price $\hat{P}$ between the separating price of a_δ and $V^M(D_\delta)$ that renders type a_δ exactly indifferent between announcing its separating dividend and announcing $\hat{D}$. In particular,

$$\Lambda_s\left(a_\delta\right) + \frac{1}{1+i}F\left(a_\delta - \Lambda_s\left(a_\delta\right)\right) < \hat{V}_1 < D_\delta + \frac{1}{1+i}E\left(F\left(x - D_\delta|D_\delta\right)\right). \tag{25}$$

We claim now that there is also unique set of concentrated beliefs that corresponds to this market price. Namely, there is a unique $\hat{x} \in (a_\delta, c_\delta)$ such that $\hat{V}_1 = \hat{V}_1\left(\hat{x}, \hat{D}\right) = \hat{D} + \frac{1}{1+i}F\left(\hat{x} - \hat{D}\right)$.

First note that for $\hat{x} = a_\delta$,

$$\hat{D}+\frac{1}{1+i}F\left(\hat{x}-\hat{D}\right)=\hat{D}+\frac{1}{1+i}F\left(a_\delta-\hat{D}\right)<\Lambda_s\left(a_\delta\right)+\frac{1}{1+i}F\left(a_\delta-\Lambda_s\left(a_\delta\right)\right), \quad (26)$$

where the inequality follows since $F' > 1+i$ on the relevant range.

Also, if $\hat{x} = c_\delta$ then

$$\begin{aligned}\hat{D}+\frac{1}{1+i}F\left(\hat{x}-\hat{D}\right) &= \hat{D}+\frac{1}{1+i}F\left(c_\delta-\hat{D}\right)>D_\delta+\frac{1}{1+i}F\left(c_\delta-D_\delta\right) \quad (27)\\ &= D_\delta+\frac{1}{1+i}E\left(F\left(x-D_\delta|D_\delta\right)\right),\end{aligned}$$

where the inequality follows again by $F' > 1+i$, and the last equality from (24).

Combining (25), (26), and (27) we obtain that by varying $\hat{x}$, $\hat{V}_1\left(\hat{x},\hat{D}\right)$ can take all possible values between the separating price of a_δ and the price corresponding to the smoothed dividend D_δ. Furthermore, since $F(\cdot)$ is strictly increasing, there is a unique $\hat{x} = \hat{x}\left(\hat{D}\right)$ that corresponds to the market price $\hat{V}_1$.

Consider now an out-of-equilibrium dividend $\hat{D} \in \left(b_\delta - I^{FB} + \delta, b_\delta - I^*\right)$. Compare the utility of a type b_δ manager who issues a dividend $\hat{D}$ to the utility of such a manager if he issues the separating dividend. If beliefs are such that the market price is identical to the separating price, then the manager obviously prefers $\hat{D}$ because the market price is the same while underinvestment is lower under $\hat{D}$. If on the other hand, beliefs are such that the market price is equal to $V^M(D_\delta)$ (Eq. 22), then the type b_δ manager strictly prefers the separating dividend. This is so because the market price under $\hat{D}$ is identical now to the smoothed dividend price, while the intrinsic value is higher in the smoothed dividend case, and we know that a type b_δ is indifferent between separation and smoothing. By continuity, there is a market price $\hat{V}_2$ between the separating price of b_δ and $V^M(D_\delta)$ that renders type b_δ exactly indifferent between announcing its separating dividend and announcing $\hat{D}$. In particular,

$$D_\delta+\frac{1}{1+i}E\left(F\left(x-D_\delta|D_\delta\right)\right)<\hat{V}_2<\Lambda_s\left(b_\delta\right)+\frac{1}{1+i}F\left(b_\delta-\Lambda_s\left(b_\delta\right)\right) \quad (28)$$

We claim now that there is also unique concentrated belief that correspond to this market price. Namely, there is a unique $\hat{x} \in (c_\delta, b_\delta)$ such that $\hat{V}_2 = \hat{V}_2\left(\hat{x},\hat{D}\right) = \hat{D} + \frac{1}{1+i}F\left(\hat{x}-\hat{D}\right)$.

First note that for $\hat{x} = b_\delta$,

$$\hat{D}+\frac{1}{1+i}F\left(\hat{x}-\hat{D}\right)=\hat{D}+\frac{1}{1+i}F\left(b_\delta-\hat{D}\right)>\Lambda_s\left(b_\delta\right)+\frac{1}{1+i}F\left(b_\delta-\Lambda_s\left(b_\delta\right)\right) \quad (29)$$

where the inequality follows since $F' > 1 + i$ on the relevant range.

Also, if $\hat{x} = c_\delta$ then

$$\hat{D} + \frac{1}{1+i} F\left(\hat{x} - \hat{D}\right) = \hat{D} + \frac{1}{1+i} F\left(c_\delta - \hat{D}\right) < D_\delta + \frac{1}{1+i} F\left(c_\delta - D_\delta\right) \quad (30)$$
$$= D_\delta + \frac{1}{1+i} E\left(F\left(x - D_\delta | D_\delta\right)\right),$$

where the inequality follows again by $F' > 1 + i$, and the last equality from (24).

Combining (28), (29), and (30) we obtain that by varying $\hat{x}$, $\hat{V}_2\left(\hat{x}, \hat{D}\right)$ can take all possible values between the separating price of b_δ and the price corresponding to the smoothed dividend D_δ. Furthermore, since $F(\cdot)$ is strictly increasing, there is a unique $\hat{x} = \hat{x}\left(\hat{D}\right)$ that corresponds to the market price $\hat{V}_2$.

We have proved the following result:

Lemma 11 *The following holds:*

1. *For each out-of-equilibrium dividend $\hat{D} \in \left(a_\delta - I^*, b_\delta - I^{FB} + \delta\right)$ there is a unique $\hat{x} = \hat{x}\left(\hat{D}\right)$ such that if beliefs given $\hat{D}$ are that the manager's type is $\hat{x}$ with probability 1, then type a_δ is indifferent between announcing its separating dividend and $\hat{D}$.*

2. *For each out-of-equilibrium dividend $\hat{D} \in \left(b_\delta - I^{FB} + \delta, b_\delta - I^*,\right)$ there is a unique $\hat{x} = \hat{x}\left(\hat{D}\right)$ such that if beliefs given $\hat{D}$ are that the manager's type is $\hat{x}$ with probability 1, then type b_δ is indifferent between announcing its separating dividend and $\hat{D}$.*

From the derivation above, it is easy to see that $\hat{x}\left(\hat{D}\right)$ is continuous at $D_\delta = b_\delta - I^{FB} + \delta$. Indeed, as $\hat{D} \to D_\delta$ types a_δ and b_δ can be kept indifferent if and only if $\hat{x}\left(\hat{D}\right) \to c_\delta$. Moreover, it is also straightforward to verify that $\hat{x}\left(\hat{D}\right)$ is strictly increasing. To see this assume for example that type a_δ is indifferent between announcing its separating dividend and two out-of-equilibrium dividends $\hat{D}_1 < \hat{D}_2$. Since underinvestment is higher given $\hat{D}_2$ it must be that the corresponding price is also higher to maintain the indifference. Thus: $\hat{x}\left(\hat{D}_2\right) > \hat{x}\left(D_1\right)$. We have proved the following result:

Proposition 4 *Let $\delta \in [\underline{\Delta}, \Delta^*)$. There is a unique set of concentrated out-of-equilibrium beliefs such that:*

1. *For each out-of-equilibrium dividend* $\hat{D} \in \left(a_\delta - I^*, b_\delta - I^{FB} + \delta\right)$, *type* a_δ *is indifferent between announcing* $\hat{D}$ *and announcing its separating dividend* $\Lambda_s\left(a_\delta\right)$.

2. *For each out-of-equilibrium dividend* $\hat{D} \in \left(b_\delta - I^{FB} + \delta, a_\delta - I^*\right)$, *type* b_δ *is indifferent between announcing* $\hat{D}$ *and announcing its separating dividend* $\Lambda_s\left(b_\delta\right)$.

Furthermore, this set of beliefs is continuous and monotone increasing.

We can now replace the out-of-equilibrium beliefs used in Section 3 with the monotone out-of-equilibrium beliefs derived in Proposition 4. The existence result is then intact as are all of the other results in the paper.